Answer Key

1

(a) $x^2 + 2xy + y^2 = 3$

$x^2 + y^2 = 3 - 2xy$

(b) $(3x + 2y)^2 + 5y^2$

$= 9x^2 + 12xy + 4y^2 + 5y^2$

$= 9x^2 + 9y^2 + 12xy$

$= 9(x^2 + y^2)$

$= 9(3 - 2xy)$

$= 27 - 18xy + 12xy$

$= 27 - 6xy$

$= 27 - 14$

$= 13$

2

(a) $\frac{30+5x-2}{6} = x$

$28 + 5x = 6x$

$x = 28$

(b) $\frac{9a^2+b^2}{(3a+b)(3a-b)} - \frac{3a-b}{3a+b}$

$= \frac{9a^2+b^2-(3a+b)(3a-b)}{(3a+b)(3a-b)}$

$= \frac{9a^2+b^2-9a^2+6ab-b^2}{(3a+b)(3a-b)}$

$= \frac{6ab}{(3a+b)(3a-b)}$

3

$$a = c + \sqrt{\frac{2(b^2+1)}{a}}$$

$$a(a-c)^2 = 2b^2 + 2$$

$$b^2 = \frac{a(a-c)^2 - 2}{2}$$

$$b = \pm\sqrt{\frac{a(a-c)^2 - 2}{2}}$$

4

(a) $A = \frac{20000}{x}$ min

(b) $A = \frac{20000}{x-8}$ min

(c) $\frac{20000}{x-8} - \frac{20000}{x} = 120$

$\frac{20000x-20000(x-8)}{x(x-8)} = 120$

$\frac{160000}{x(x-8)} = 120$

$120x(x-8) = 160000$

$120x^2 - 960x - 160000 = 0$

$3x^2 - 24x - 4000 = 0$ (shown)

(d) $x = \frac{-(-24)\pm\sqrt{(-24)^2-4(3)(-4000)}}{2(3)}$

$= \frac{-(-24)\pm\sqrt{48576}}{6}$

$= 40.73327\ or\ -32.73327$

$= 40.73\ (2dp) or\ -32.73\ (2dp)$

(e) $time = \frac{20000}{x+x-8}$

$= \frac{20000}{2x-8}$

$= \frac{20000}{2(40.73327\ldots)-8}$

$= 272.23277..mins$

$= 4h\ 32mins$ (nearest minute)

5

$\left(\frac{8p^6}{27m^3}\right)^{-\frac{1}{3}}$

$= \left(\frac{27m^3}{8p^6}\right)^{\frac{1}{3}}$

$= \frac{27^{\frac{1}{3}}m}{8^{\frac{1}{3}}p^2}$

$= \frac{3m}{2p^2}$

6

(a) Ley r cm can be the radius

$(r+4)^2 = r^2 + 16^2$

$r^2 + 8r + 16 = r^2 + 256$

$r = 30$

(b) $\tan \angle ROQ = \frac{16}{30}$

$= 0.490\ rad\ (3sf)$

(c) Arc length $PQ = 30(0.48995)$

Perimeter $= 30(0.48995) + 16 + 4$

$34.7\ cm\ (3\ sf)$

(d) Area of sector OPQ

$= \frac{1}{2}(30^2)(0.48995)$

Shaded area

$= \frac{1}{2}(30)(16) - \frac{1}{2}(30^2)(0.48995)$

7

(a) $AO = OB\ (radii\ of\ circle)$

$\angle TAO = \angle TBO = 90°$

(radius perpendicular to tangent)

$OT = OT$

(common side, hypotenuse)

By RHS, triangle AOT is congruent to triangle BOT

(b) (i) $\angle CAO = 90° - 58° = 32°$

(radius perpendicular to tangent)

(ii) $\angle AOB = 50° \times 2 = 100°$

(angle at center = 2x angle at circumference)

(iii) $\angle ATB$

$= 360° - \angle TAO - \angle TBO - obtuse\angle AOB$

$= 360° - 90° - 90° - 100° = 80°$

(property of quadrilateral ATBO)

OR

$\angle AOT = 100° \div 2 = 50°$

$\angle ATO = 180° - 90° - 50°$

(sum of angles of a triangle)

$\angle ATB = 40° \times 2 = 80°$

(c) $\angle ACO = \angle CAO = 32°$

(isosceles triangle AOC)

$\angle OCB = \angle ACB - \angle ACO$

$= 50° - 32° = 18°$

$\angle OBC = \angle OCB = 18°$

(isosceles triangle COB)

(d) By angles in opposite segment property, D is a point on the circle.

8 (a) Area of triangle ABC

$= \frac{1}{2}(95)(82)\ sin\ 75°$

$= 3760\ cm^3 (3\ s.f)$

(b) $AC^2 = 95^2 + 82^2 - 2(95)(82)cos75°$

$AC = 108.24$

$= 108\ m\ (3\ s.f)$

(c) $\frac{\sin \angle ACB}{95} = \frac{\sin 75°}{108.24}$

$\angle ACB = 57.970°$

$= 58.0°$

(d)

$\angle DAC = 57.970° - 25°$

$= 32.979°$ (alt. angles, parallel lines)

$$\sin 32.97° = \frac{DE}{71}$$

$$DE = 38.638\ m$$

Let the greatest angle of elevation be θ.

$$\tan \theta = \frac{67}{38.638}$$

$$38.638$$

$$\theta = 60.0° (1\ dp)$$