Name:	School:	Target Grade:

PARADIGM

SECONDARY 3 AM WA2

MOCK EXAM PAPER

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Find a nice comfortable spot without distraction.

2. Be fully focused for the whole duration of the test.

3. Speed is KING. Finish the paper as soon as possible then return-back to Check Your Answers.

4. As you are checking your answers, always find ways to VALIDATE your answer.

5. Avoid looking through line by line as usually you will not be able to see your Blind Spot.

6. If there is no alternative method, cover your answer and REDO the question.

7. Give non-exact answers to 3 significant figures, or 1 decimal place for angles in degree, or 2 decimal place for $$$, unless a different level of accuracy is specified in the question.

Wish you guys all the best in this test.

You can do it.

I believe in you.

Team Paradigm

If you are struggling in this paper, it's an indication to work harder!
If you need support and personalised guidance, you can find us here
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PARADIGM [Turn Over]

Name: ______________________________ Class: ______ Date: _________

Secondary 3 Mathematics
WA2 Mock Paper

Topic:

Duration: 1 hour 40 minutes

65

Surds

1	Given that $\sqrt{p + q\sqrt{8}} = \frac{9}{4-\sqrt{8}}$, where p and q are rational numbers, find the values of p and q.	[4]
2	An open cuboid bin has a square base of side $(\sqrt{7} - \sqrt{5})$ m. The capacity of the bin is $(90\sqrt{5} - 76\sqrt{7})$ m^3. Find the exact value of **(a)** the base area of the bin, **(b)** the height of the bin, **(c)** the total surface area of the bin.	 [2] [3] [2]
3	Without using a calculator, find the integer value of a and of b for which the solution of the equation $2x\sqrt{5} = x\sqrt{2} + \sqrt{18}$ is $\frac{\sqrt{a}+b}{3}$.	[3]

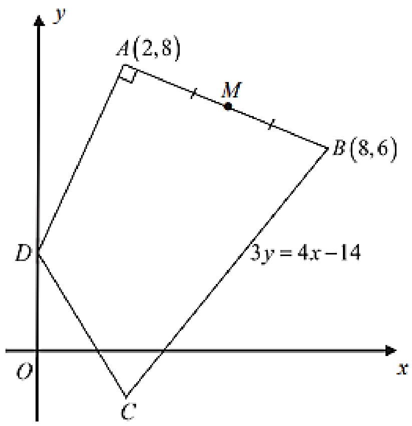
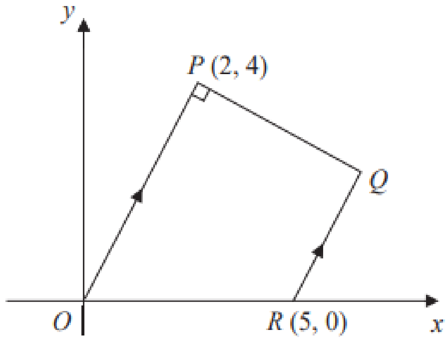
Polynomial

1	The polynomial $2x^3 - 3ax^2 - 2ax + b$ has a factor $x - 12$ and leaves a remainder of –8 when divided by $x - 1$. (a) Find the values of a and of b. (b) Using the values of a and b in part (a), factorise the polynomial completely.	 [4] [3]
2	The term containing the highest power of x in the polynomial $f(x)$ *is* $3x^4$. Two of the roots of the equation $f(x) = 0$ *are* $x = -1$ *and* $x = k$ where k in an integer. Given that $x^2 - 2x + 6$ is a quadratic factor of $f(x)$ *and* $f(x)$ leaves a remainder of – 36 when divided by x, (a) show that $k = 2$. Hence, (b) determine the number of real roots of the equation $f(x) = 0$.	 [3] [3]
3	A polynomial, $P(x)$, *is* $2x^2 + ax^2 + 6x + 27$, where a is a constant. The quadratic expression $x^2 + bx + 9$ is a factor of $P(x)$, where b is a constant. (a) Find the remaining factor of $P(x)$. (b) Show that the equation $P(x) = 0$ has only one real root.	 [2] [3]

Partial Fractions

1	Express $\frac{3x^3+6x-8}{x(x^2+2)}$ in partial fractions.	[7]
2	Express $\frac{x^2+2x-19}{(x-1)(x+3)^2}$ as a sum of three partial fractions.	[5]

Coordinate Geometry

1	The diagram shows a quadrilateral *ABCD*. The coordinates of *A* and *B* are $(2,8)$ and $(8,6)$ respectively. *M* is the midpoint of *AB* and *CM* is perpendicular to *AB*. The equation of *BC* is $3y = 4x - 14$. The point *D* lies on the *y*-axis and $DAB = 90^{\circ}$.  (i) Find the coordinates of *D*. (ii) Find the coordinates of *C*. (iii) Find the coordinates of *ABCD*.	[3] [5] [2]
3	Solutions to this question by accurate drawing will not be accepted. In the trapezium *OPQR*, the point *P* has coordinates $(2,4)$ and the point *R* has coordinates $(5,0)$. The sides *OP* and *QR* are parallel, and *PQ* is perpendicular to *OP*.  (i) Show that the coordinates of *Q* are $(6,2)$ (ii) Find the equation of the perpendicular bisector of *OP* and explain if the perpendicular bisector cuts the line segment *QR*. (iii) *T* is a point which lies on the perpendicular bisector of *OP* such that the area of quadrilateral *ORPT* IS 25 units^2. Find the coordinates of *T*.	[5] [3] [3]

Answer Key

Surds

1	Solutions: $\sqrt{p+q\sqrt{8}}=\frac{9}{4-\sqrt{8}}$ $p+q\sqrt{8}=\left(\frac{9}{4-\sqrt{8}}\right)^2$ $=\frac{81}{24-8\sqrt{8}}$ $=\frac{81}{24-8\sqrt{8}}\times\frac{24+8\sqrt{8}}{24+8\sqrt{8}}$ $=\frac{81(24+8\sqrt{8})}{64}$ $=\frac{243}{8}+\frac{81}{8}\sqrt{8}$ $p=\frac{243}{8}$ or $q=\frac{81}{8}$ Ans: $p=\frac{243}{8} q=\frac{81}{8}$	
2	Solutions: (i) Area= $\left(\sqrt{7}-\sqrt{5}\right)^2=12-2\sqrt{35}$ (ii) Height $=\frac{(90\sqrt{5}-76\sqrt{7})}{(12-2\sqrt{35})}\times\frac{(12+2\sqrt{35})}{(12+2\sqrt{35})}$ $=\frac{1080\sqrt{5}+180\sqrt{175}-912\sqrt{7}-152\sqrt{245}}{144-4\times35}$ $=\frac{1080\sqrt{5}+180\times5\sqrt{7}-912\sqrt{7}-1064\sqrt{5}}{4}$ $=\frac{16\sqrt{5}-12\sqrt{7}}{4}=4\sqrt{5}-3\sqrt{7}$ (iii)Total Surface area = $12-2\sqrt{35}+4\left(\sqrt{7}-\sqrt{5}\right)\left(4\sqrt{5}-3\sqrt{7}\right)$ $=12-2\sqrt{35}+28\sqrt{35}-164$ $=26\sqrt{35}-152$ Ans: (i) $12-2\sqrt{35}$ (ii) $4\sqrt{5}-3\sqrt{7}$ (iii) $26\sqrt{35}-152$	
3	Solution: $x\left(2\sqrt{5}-\sqrt{2}\right)=\sqrt{18}$ $x=\frac{\sqrt{18}}{2\sqrt{5}-\sqrt{2}}\times\frac{2\sqrt{5}+\sqrt{2}}{2\sqrt{5}+\sqrt{2}}$ $=\frac{2\sqrt{90}+6}{18}$ $=\frac{6\sqrt{10}+6}{18}$ $=\frac{\sqrt{10}+1}{3}$ $a=10,\quad b=1$ Ans: $a=10,\quad b=1$	

Polynomial

1 Solutions:

(a) $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 - 3a\left(\frac{1}{2}\right)^3 - 2a\left(\frac{1}{2}\right)$

$\frac{1}{4} - \frac{3a}{4} - a + b = 0$

$7a - 4b = 1$ --- (1)

$f(1) = 2 - 3a - 2a + b = -8$

$5a - b = 10$

$b = 5a - 10$ --- (2)

Sub (2) in (1):

$7a - 4(5a - 10) = 1$

$7a - 20a + 40 = 1$

$-13a + 39 = 0$

$a = 3$

$b = 5(3) - 10$

$= 5$

(b)

$$
\begin{array}{r|l}
 & x^2 \quad - 4x \quad 5 \\
\hline
2x - 1 & 2x^3 - 9x^2 - 6x + 5 \\
 & - \\
 & 2x^3 \quad -z^2 \\
\hline
 & 8x^2 \quad\quad - 6x + 5 \\
 & - \\
 & 8x^2 \quad\quad + 4x \\
\hline
 & -10x + 5 \\
 & - \\
 & -10x + 5 \\
\hline
 & 0
\end{array}
$$

Answer: (a) 5 (b) $(2x-1)(x-5)(x-1)$

2 Solutions:

(a) $\mathrm{f}(x) = 3(x^2 - 2\mathrm{x} + 6)(x+1)(x-k)$

$\mathrm{f}(0) = -36$

$3(6)(-1)(k) = -36$

$k = 2$ (shown)

(b) $x^2 - 2x + 6 = 0$

Discriminant $= (-2)^2 - 4(6) < 0$

∴ no real roots

When $f(x) = 0$, there are 2 real roots

Answer: (a) $k = 2$ Shown (b) When $f(x) = 0$, there are 2 real roots.

3 Solutions:

(a) Let $2x^3 + ax^2 + 6x + 27 = (x^2 + bx + 9)(cx + d)$

Compare coefficient of x^3,

$c = 2$

Compare constant,

$9d = 27$

$d = 3$

$P(x) = 2x + 3$

(b) $2x^3 + ax^2 + 6x + 27 = (x^2 + bx + 9)(2x + 3)$
Compare coefficient of x^2,
$a = 3 + 2b$ ------------------ (1)
Compare coefficient of x,
$6 = 3b + 18$
$b = -4$ ------------------------ (2)
Sub (2) into (1),
$a = 3 + 2(-4)$
$a = -5$
$2x^3 + 5x^2 + 6x + 27 = 0$
$(x^2 - 4x + 9)(2x + 3) = 0$
$x^2 - 4x + 9 = 0$ or $2x + 3 = 0$
For $x^2 = 4x + 9 = 0$,
Discriminant $= (-4)^2 - 4(1)(9)$
$= -20$
Hence there is no solution for $x^2 = 4x + 9 = 0$.
$2x + 3 = 0$
$x = -\frac{3}{2}$
Hence the equation $P(x) = 0$ has only one real root

Answer: (a) $P(x) = 2x + 3$ (b) $x = -\frac{3}{2}$

Partial Fractions

1 Solution:
By long division,

$$\begin{array}{r|l} & \quad\quad\quad\quad 3 \\ \hline x^2 + 2x & 3x^2 + 6x - 8 \\ & -(3x^2 + 6x) \\ & -8 \end{array}$$

$\frac{3x^3+6x-8}{x(x^2+2)} = 3 - \frac{8}{x(x^2+2)}$
$\frac{-8}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2}$
$8 = A(x^2 + 2) + (Bx + C)(x)$

Using substitution, $x = 0$,
$-8 = 2A$
$A = -4$
Comparing x-coefficient,
$C = 0$
Comparing x^2-coefficient,
$0 = A + B$
$B = 4$
Therefore,
$\frac{3x^3+6x-8}{x(x^2+2)} = 3 - \frac{4}{x} + \frac{4x}{(x^2+2)}$

Answer: $3 - \frac{4}{x} + \frac{4x}{(x^2+2)}$

2	Solution:
	$$\frac{x^2+2x-19}{(x-1)(x+3)^2}=\frac{A}{(x-1)}+\frac{B}{x+3}+\frac{C}{(x+3)^2}$$
	$$x^2+2x-19=A(x+3)^2+B(x-1)(x+3)+C(x-1)$$
	Sub $x=1, -16=16A$
	$A=-1$
	Sub $x=-3, -16=-4C$
	$C=4$
	Sub $x=0, -19=9A-3B-C$
	Sub $A=-1, C=4, -19=-9-3B-4$
	$B=2$
	$$\frac{x^2+2x-19}{(x-1)(x+3)^2}=-\frac{1}{(x-1)}+\frac{2}{(x+3)}+\frac{4}{(x+3)^2}$$
	Answer: $-\frac{1}{(x-1)}+\frac{2}{x+3}+\frac{4}{(x+3)^2}$	

Coordinate Geometry

1	Solutions: (i) Let $D(0,a)$ $M_{AB}=-\frac{1}{3}$ $M_{AD}=3$ $\frac{8-a}{2-0}=3$ $a=2$ $D(0,2)$ (ii) $y=5x+c$ At $(5,7), c=-8$ $y=3x-8$ $y=\frac{4}{3}x-\frac{14}{3}$ $\frac{4}{3}x-\frac{14}{3}=3x-8$ $x=2$ $y=-2$ $C(2,-2)$ (iii) $\frac{1}{2}\begin{vmatrix}2 & 8 & 2 & 0 & 2\\ 8 & 6 & -2 & 2 & 8\end{vmatrix}$ $=\frac{1}{2}\lvert(12-16+4)-(64+12+4)\rvert$ $=40$ units^2 Ans: (i) $D(0,2)$ (ii) $C(2,-2)$ (iii) 40 units^2	
3	Solutions: (i) Gradient of $OP=\frac{4}{2}$ $=2$ Equation of PQ: $4=-\frac{1}{2}(2)+c$ $c=5$ Equation of PQ is $y=-\frac{1}{2}x+5$ ------ (1) Equation of QR: $0=2(5)+c$ $c=-10$	

	Equation of QR is $y = 2x - 10$ ------ (2) Sub (1) into (2): $-\frac{1}{2}x + 5 = 2x - 10$ $x = \frac{15}{2.5}$ $= 6$ Sub $x = 6$ into (2): $y = 12 - 10$ $= 2$ Thus, coordinates of Q is $(6, 2)$. (shown) (ii) Midpoint of $OP = \left(\frac{0+2}{2}, \frac{0+4}{2}\right)$ $= (1,2)$ Equation of perpendicular bisector of OP: $2 = -\frac{1}{2}(1) + c$ $c = 2\frac{1}{2}$ Equation of perpendicular bisector of OP is $y = -\frac{1}{2}x + 2\frac{1}{2}$ --------- (3) Sub (3) into (2): $-\frac{1}{2}x + 2\frac{1}{2} = 2x - 10$ $x = \frac{12.5}{2.5}$ $= 5$ Sub $x = 5$ into (2): $y = 2(5) - 10$ $= 0$ Thus the perpendicular bisector of OP cuts the line segment QR at $R(5,0)$. Ans: (i) Sub (1) into (2) $=$ 6, Sub $x = 6$ into (2) $= 2$, (ii) Sub (3) into (2) $= 5$, Sub $x = 5$ into (2) $= 0$, (iii) $x =- 5$, $y = 5$. Coordinates of T is $(-5a, 5)$	