| Name: | School: | Target Grade: |
| :--- | :--- | :--- |

## SECONDARY 3 A Math WA1 <br> MOCK EXAM PAPER

## READ THESE INSTRUCTIONS FIRST <br> INSTRUCTIONS TO CANDIDATES

1. Find a nice comfortable spot without distraction.
2. Be fully focused for the whole duration of the test.
3. Speed is KING. Finish the paper as soon as possible then return-back to Check Your Answers.
4. As you are checking your answers, always find ways to VALIDATE your answer.
5. Avoid looking through line by line as usually you will not be able to see your Blind Spot.
6. If there is no alternative method, cover your answer and REDO the question.
7. Give non-exact answers to 3 significant figures, or 1 decimal place for angles in degree, or 2 decimal place for $\$ \$ \$$, unless a different level of accuracy is specified in the question.

Wish you guys all the best in this test.
You can do it.

I believe in you.
Team Paradigm

If you are struggling in this paper, it's an indication to work harder! If you need support and personalised guidance, you can find us here www.mathtutor.com.sg


Name: $\qquad$

## Secondary 3 Additional Mathematics WA1 Mock Paper

Topic: Quadratic Functions, Equations \& Inequalities
Duration: 60 mins

## Quadratic Functions

Class:
$\qquad$ Date: $\qquad$

1 (a) A dolphin jumps out of the water during a dolphin show. The height, $y \mathrm{~m}$, of the dolphin is given by $y=-4.5 t^{2}+9 t$, where $t$ is the time in seconds after the dolphin jumps out of the water.
(i) Express $y=-4.5 t^{2}+9 t$ in the form $y=a(t-h)^{2}+k$.
(ii) How long did it take the dolphin to reach the maximum height and what is the maximum height?
(iii) Find the range of values of $t$ where the height of the dolphin is at least

$$
3.375 \mathrm{~m} .
$$

## Quadratic Inequalities

| 1 | Find the set of values of x for which $(2+3 x)(x-5)>2+3 x$. | $[3]$ |
| :--- | :--- | :--- |
| 2 | Find the range of values of $x$ where the curve $y=\frac{6-x^{2}}{4 x^{2}-12 x+9}$ is below the $x$-axis. | $[4]$ |

## Nature of Roots (Finding Unknown Value)

| 1 | (a) The equation of a curve is $y=p x^{2}-4 x+16 p$. Find the range of values <br> of $p$ given that the curve lies completely above the x -axis. <br> (b) Find the value of $h$ for which the line $y=2 x+h$ is a tangent to the curve <br> $y=2 x^{2}-6 x+5$. | $[4]$ |
| :--- | :--- | :--- |
| 2 | Determine the range of values of k such that the curve <br> $y=3 x^{2}+k^{2}-k-3$ intersects the line $y=3 k x$ at two distinct points. | $[4]$ |

## Nature of Roots (Proving)

(a) Show that the equation $x^{2}+2(m-1) x+(2 m-3)=0$ has real roots for all real values of $x$.
(b) Explain why the quadratic expression $-x^{2}+3 x-4$ is always negative.

## Simultaneous Equation

1 Find the coordinates of the points of intersection of the curve $x^{2}-x y+y^{2}=16$ and the line $2 x-3 y=4$.

## Answer Key

## Quadratic Functions

(i) $y=4.5 t^{2}+9 t$

$$
\begin{aligned}
& =-4.5\left(t^{2}+2 t\right) \\
& =-4.5\left(t^{2}+2 t+1-1\right) \\
& =-4.5(t-1)^{2}+4.5
\end{aligned}
$$

(ii) Time taken to reach maximum height $=1$ second Maximum Height $=4.5 \mathrm{~m}$
(iii) $-4.5 t^{2}+9 t \geq 3.375$

$$
4.5 t^{2}-9 t+3.375 \leq 0
$$

$$
4 t^{2}-8 t+3 \leq 0
$$

$$
(2 t-3)(2 t-1) \leq 0
$$

$$
\frac{1}{2} \leq t \leq 1 \frac{1}{2}
$$

Ans: (i)-4.5 $(t-1)^{2}+4.5$ (ii) 4.5 m (iii) $\frac{1}{2} \leq t \leq 1 \frac{1}{2}$

## Quadratic Inequalities

| 1 | $\begin{aligned} & (2+3 x)(x-5)>2+3 x \\ & (2+3 x)(x-5)-(2+3 x)>0 \\ & (2+3 x)(x-6)>0 \\ & + \\ & \hline-\frac{2}{3} \\ & \therefore x<-\frac{2}{3} \text { or } x>6 \end{aligned}$ |
| :---: | :---: |
| 2 | $\begin{aligned} & y=\frac{6-x^{2}}{4 x^{2}-12 x+9}, \quad y<0 \\ & \frac{6-x^{2}}{4 x^{2}-12 x+9}<0 \\ & \frac{6-x^{2}}{(2 x-3)^{2}}<0 \end{aligned}$ <br> Since $(2 x-3)^{2}$ is always positive if $x \neq \frac{3}{2}$, <br> Ans: $x<-\sqrt{6}$ or $x>\sqrt{6}, x<-2.45$ or $x>2.45, x \neq \frac{3}{2}$ |

Nature of Roots (Finding Unknown Value)

$$
\begin{array}{|l|l}
1 & \begin{aligned}
\text { (a) } b^{2}-4 a c & =(-4)^{2}-4(p)(16 p) \\
& =16-64 p^{2}
\end{aligned} \\
\text { Curve lies above } x \text {-axis, } b^{2}-4 a c<0 \\
16-64 p^{2}<0 \\
16<64 p^{2} \\
16\left(4 p^{2}-1\right)>0 \\
(2 p+1)(2 p-1)>0 \\
p<-\frac{1}{2} \quad, \quad p>\frac{1}{2}
\end{array}
$$

(reject as $p>0$ )
(b) $y=2 x^{2}-6 x+5$
$y=2 x+h$
$y=2 x^{2}-6 x+5---(2)$
Sub (1) in (2): $2 x+h=2 x^{2}-6 x+5$

$$
\begin{aligned}
& 2 x^{2}-6 x-2 x+5-h=0 \\
& 2 x^{2}-8 x+5-h=0
\end{aligned}
$$

Discriminant $=(-8)^{3}-4(2)(5-h)=0$

$$
\begin{aligned}
64-40+8 h & =0 \\
h & =-3
\end{aligned}
$$

Ans: (a) $p>\frac{1}{2}$
(b) $h=-3$
$2 a=3, b=-3 k, c=k^{2}-k-3$
For real roots,

$$
\begin{aligned}
& b^{2}-4 a c>0 \\
&(-3 k)^{2}-4(3)\left(k^{2}-k-3\right)>0 \\
& 9 k^{2}-12\left(k^{2}-k-3\right)>0 \\
&-3 k^{2}+12 k+36>0 \\
& k^{2}-4 k-12<0 \\
&(k-6)(k+2)<0 \\
& \underbrace{}_{-2<k<6}
\end{aligned}
$$

Ans: $-2<k<6$

## Nature of Roots (Proving)

1 (a) $b^{2}-4 a c$

$$
\begin{aligned}
& =[2(m-1)]^{2}-4(1)(2 m-3) \\
& =4 m^{2}-16 m+16 \\
& =4\left(m^{2}-4 m+4\right) \\
& =4(m-2)^{2}
\end{aligned}
$$

Since $(m-2)^{2} \geq 0$

$$
\begin{aligned}
4(m-2)^{2} & \geq 0 \\
b^{2}-4 a c & \geq 0
\end{aligned}
$$

The equation has real roots for all values of $x$.
(b) $b^{2}-4 a c=3^{2}-4(-1)(-4)$

$$
\begin{aligned}
& =9-16 \\
& =-7<0, \text { so, no real roots and does not intersect } x \text {-axis. }
\end{aligned}
$$

$a=-1<0$, so, the shape is
And, $y=-x^{2}+3 x-4$ lies entirely below the $x$-axis.


Hence $-x^{2}+3 x-4$ is always negative.

## Simultaneous Equation

```
\(1 \quad x^{2}-x y+y^{2}=16\)---- (1)
    \(2 x-3 y=4\)
    \(x=\frac{4+3 y}{2}\)

Substitute (2) into (1)
\(\left(\frac{4+3 y}{2}\right)^{2}-y\left(\frac{4+3 y}{2}\right)+y^{2}=16\)
\(7 y^{2}+16 y-48=0\)
\((7 y-12)(y+4)=0\)
\(y=1 \frac{5}{7}\) or -4
\(x=4 \frac{4}{7}\) or -4
The coordinates of the points of intersections are \(\left(4 \frac{4}{7}, 1 \frac{5}{7}\right)\) and \((-4,-4)\).
Ans: \(\left(4 \frac{4}{7}, 1 \frac{5}{7}\right)\) and \((-4,-4)\)```

