MUST KNOW QUESTIONS TO CONQUER

TRIGONOMETRY

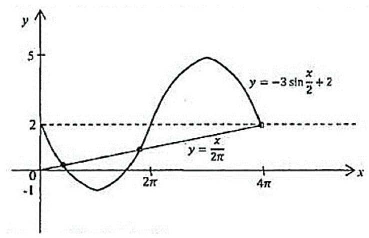
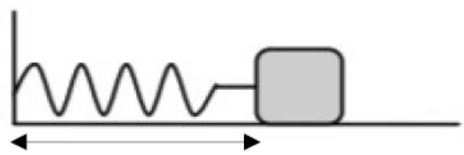
Simplifying Trigonometry

1	Without using a calculator, (i) show that $\sin 105° = \frac{1+\sqrt{3}}{2\sqrt{2}}$, (ii) hence, express $1 + \cot^2 105°$ in the form $a + b\sqrt{3}$, where a and b are integers. Ans: (i) $\frac{1+\sqrt{3}}{2\sqrt{2}}$ (ii) $8 - 4\sqrt{3}$
2	**(a)** State the values between which each of the following must lie: **(i)** the principal value of $\tan^{-1} x$, **(ii)** the principal value of $\cos^{-1} x$. **(b)** Without using a calculator, find the **exact** value of $\tan 105°$. Answers: (a)(i) $-90° < \tan^{-1} x < 90°$ (ii) $0° \leq \cos^{-1} \leq 180°$ (b) $\tan 105° = -\sqrt{3} - 2$
3	Two acute angles, A and B are such that $\cot A = 7$ and $\tan(A - B) = -1$. Without evaluating A and B, (i) show that $\tan B = \frac{4}{3}$, (ii) evaluate $\sin A$ and $\cos B$, (iii) evaluate $\sin^2 2A + \cos^2 2B$. Ans: (ii) $\sin A = \frac{1}{\sqrt{50}}$, $\cos B = \frac{3}{5}$ (iii) $\frac{98}{625}$

Trigonometry (Quadrants)

1	Given that $\sin A = -p$ and $\cos B = -q$, where A and B are in the same quadrant and p and q are positive constants, find the value of (i) $\sin(-A)$, (ii) $\tan(45° - A)$, (iii) $\sec(2B)$. Ans: (i) $\sin(-A) = p$ (ii) $\tan(45° - A) = \frac{\sqrt{1-p^2}-p}{\sqrt{1-p^2}+p}$ (iii) $\sec(2B) = \frac{1}{2q^2-1}$
2	Given that $\sin\theta = p$ where θ is an acute angle measured in degrees, obtain an expression, in terms of p, for (i) $\tan\theta$, (ii) $\sin(90° - \theta)$. Ans: (i) $\tan\theta = \frac{p}{\sqrt{1-p^2}}$ (ii)$\cos\theta = \sqrt{1-p^2}$
3	Given that θ is obtuse and$\tan\theta = a$, express, in terms of a, (i) $\cos\theta$ (ii) $\text{cosec}\,\theta$. Ans: (i) $\cos\theta = -\frac{1}{\sqrt{1+a^2}}$ (ii) $\text{cosec}\,\theta = -\frac{\sqrt{1+a^2}}{a}$
4	It is given that cos A =- m, *where* m >*0*, and that A is obtuse. Find the value of each of the following in terms of m. (i) $\tan A$ (ii) $\cot(180 - A)$ (iii) $\cos\left(\frac{A}{2}\right)$ Ans: (i) $\frac{-\sqrt{1-m^2}}{m}$ (ii) $\frac{m}{\sqrt{1-m^2}}$ (iii) $\frac{\sqrt{1-m}}{2}$

Trigonometry Graph

1	Given that $y = a + b\cos 4x$, where a and b are integers, and x is in radians, **(i)** state the period of y. Given that the maximum and minimum values of y are 3 and -5 respectively, find **(ii)** the amplitude of y, **(iii)** the value of a and of b. Answers: (i) $\frac{\pi}{2}$ (ii) amplitude = 4 (iii) $b = 4$ $a = -1$
2	The function f is given by $f(x) = -3\sin\frac{x}{2} + 2$. **(i)** State the amplitude and period of f. **(ii)** Sketch the graph of $y = f(x)$ for $0 \le x \le 4\pi$. By drawing a suitable straight line on the same axes, state the number of solutions to the equation $4\pi - x - 6\pi\sin\frac{x}{2} = 0$ for $0 \le x \le 4\pi$. Answers: (i) Amplitude = 3, Period = 4π (ii) 3 solutions 
3	An object is connected to the wall with a spring that has a original horizontal length of 20 cm. The object is pulled back 8 cm past the original length and released. The object completed 4 cycles per second.  (i) Given that the function $x = 8\cos(a\pi t) + b$, where x is the horizontal distance, in centimetres, of the object from the wall and t is the time in seconds after releasing the object, find the values of a and b. (ii) Find the duration of time for each cycle such that the object is more than 27 cm from the wall. Ans: (i) $a = 8, b = 20$ (ii) 0.0402

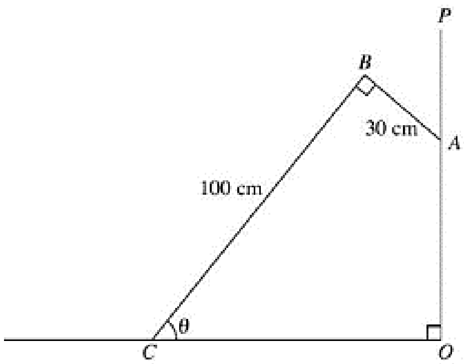
Solving Trigonometry

1	Solve the equation $6\sin^2 x + 5\cos x = 5$ for $0° < x < 360°$. Ans: $x = 99.6°,\ 260.4°$
2	Solve $\cos(2y - 80°) = \sin 42°$ for $-180° \leq y \leq 180°$ Ans: $y = 64°, 16°, -116°, -164°$
3	Solve $tan\theta = \tan(-2)$ for $0 < \theta < 2\pi$. Ans: $2\ rad, 5.14\ rad$
4	Solve the equation $2\cot^2 y = \operatorname{cosec} y + 1$ for $0° \leq y \leq 360°$. Ans: $y = 41.8°,\ 138.2°,\ 270°$
5	**(i)** Show that $\frac{\tan^2 x - 1}{\tan^2 x + 1} = 1 - 2\cos^2 x$. **(ii)** Hence find, for $0 \leq x \leq 5$, the values of x in radians for which $\frac{\tan^2 x - 1}{\tan^2 x + 1} = \frac{1}{2}$. Ans: (i) shown (ii) $x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}$

Proving Trigonometry

1	Prove $(\sec x - \tan x)(\operatorname{cosec} x + 1) = \cot x$
2	Prove $(\sin A + \cos A)(\sec A + cosec\ A) = 2 + \sec A\ cosec\ A$
3	Prove $\frac{\sec^2 \theta + 2\tan\theta}{(\cos\theta + \sin\theta)^2} = \sec^2\theta$
4	Prove $\frac{\sin A + \sin 2A}{1 + \cos A + \cos 2A} = \tan A$
5	Prove $\frac{1 - \cos 2x + \sin x}{\sin 2x + \cos x} = \tan x$

R Formula

1	Express $9\cos\theta + 12\sin\theta$ in the form $Rcos\,(\theta - \alpha)$, where R is positive and $0° < \alpha < 90°$. Hence, solve the equation $9\cos\theta + 12\sin\theta = 11$ for $0° \leq \theta \leq 360°$. Ans: (i) $15\cos(\theta - 53.1°)$ (ii) $\theta = 10.3°, 96.0°$ (1dp)
2	(i) Express $12\sin\theta\cos\theta - 8\cos^2\theta + 7$ in the form $A\sin 2\theta + B\cos 2\theta + C$, where A, B and C ate constants. (ii) Solve $12\sin\theta\cos\theta - 8\cos^2\theta + 7 = 0$ for $0° < \theta < 180°$. Ans: (i) $6sin2\theta - 4cos2\theta + 3$ (ii) $\theta = 4.6°, \theta = 119.1°$
3	The figure shows a stage prop *ABC* used by a member of the theatre, leaning against a vertical wall *OP*. It is given that $AB = 30$ cm, $BC = 100$ cm, $\angle ABC = \angle AOC = 90°$ and $\angle BCO = \theta$.  **(i)** Show that $OC = (100\cos\theta + 30\sin\theta)$ cm. Let *D* be foot of *B* on *OC*, let *E* be foot of *A* on *BD*. **(ii)** Express *OC* in terms of $R\cos(\theta - \alpha)$, where R is a positive constant and α is an acute angle. **(iii)** State the maximum value of OC and the corresponding value of θ. **(iv)** Find the value of θ for which $OC = 80$ cm. Ans: (i) shown (ii) $\therefore OC = 10\sqrt{109}\cos(\theta - 16.7°)$ (iii) $OC_{max} = 10\sqrt{109}$, $\theta = 16.7°$ (iv) $\theta = 56.7°$