6 MUST KNOW QUESTIONS TO CONQUER

POLYNOMIALS

1	The function f is defined by $f(x) = 4x^3 + px^2 + 5x + 2$, where p is a constant. **(i)** Given that $x - 1$ is a factor of f(x), find the value of p. **(ii)** Using the value of p found in **(i)**, **(a)** find the remainder when f(x) is divided by $2x - 3$, **(b)** factorise f(x) completely, **(c)** hence solve the equation $4(y-1)^3 + p(y-1)^2 + 5y - 3 = 0$. Answers: (i) $p = -11$ (ii)(a) Remainder $= -1.75$ (b) $(x-1)(4x+1)(x-2)$ (c) $y = 2, 3$ or $\frac{3}{4}$
2	(a) (i) Factorise $8x^3 + 27$. (ii) Hence determine, showing all necessary working, the number of real roots of the equation $8x^3 + 27 = 0$. (b) The coefficient of x^3 of a cubic polynomial, f(x), is 4 and that the roots of the equation $f(x) = 0$ are $-1, 3$ and k. Given that f(x) has a remainder of 60 when divided by -2 , find the value of k. Ans: (a) $(2x+3)(4x^2 - 6x + 9)$ (ii) $1\ real\ roots$ (iii) $k = 7$
3	A function f defined by f(x) = $2x^3 + px^2 + qx + 15$, where p and q are constants, has a factor of $x - 5$ and leaves a remainder of 12 when divided by $x + 1$. **(i)** Find the value of p and of q. **(ii)** Find the remainder when f(x) is divided by $2x - 3$. Ans: (i) $q = -8, p = -9$ (ii) -10.5
4	The polynomials $6x - x^3$ and $8 - 3x^2$ leave the same remainder when divided by $(x - m)$. Find the three possible values of m. Ans: $m = -2, 1, 4$
5	(i) By using long division, divide $2x^4 + 5x^3 - 8x^2 - 8x + 3$ by $x^2 + 3x - 1$. (ii)Factorise $2x^4 + 5x^3 - 8x^2 - 8x + 3$ completely. (iii)Hence find the exact solutions to the equation $$32p^4 + 40p^3 - 32p^2 - 16p + 3 = 0.$$ Ans: (i) Long Division, (ii) $(x^2 + 3x - 1)(2x - 3)(x + 1)$ (iii) $p = \frac{3}{4}$, $p = -\frac{1}{2}$, $p = \frac{-3 \pm \sqrt{13}}{4}$
6	The polynomial $f(x)$ leaves a remainder of -5 and 7 when divided by $x + 1$ and $x - 2$ respectively. Find the remainder when $f(x)$ is divided by $x^2 - x - 2$. Ans: $4x - 1$