7 MUST KNOW QUESTIONS TO CONQUER

NATURE OF ROOTS

1	The equation of a curve is $y = x^2 + (2k + 3)x + 4k + 6$, where k is a constant. (a) Find the range of values of k for which the curve lies entirely above the $x - axis$ for all real values of x. (b) Find the range of values of k for which the line $y = x + 2$ intersects with the curve. Ans: (a) $-\frac{3}{2} < k < \frac{5}{2}$ (b) $k \leq -1$ or $k \geq 3$
2	(a) Solve $5x + 3 > 2x^2$. (b) Prove that the line $y = x - 1$ always intersects the curve $y = nx^2 + 3x - n$ at 2 distinct points for all real values of n. Ans: (a) $-0.5 < x < 3$, (b) Proven
3	(a) The line $y = mx + c$ is a tangent to the circle $x^2 + y^2 = 4$, where m and c are constants. Prove that $c^2 - 4m^2 = 4$. (b) Find the value of b for which $-2 < x < \frac{1}{3}$ is the solution of $3x^2 + 5x < b$. Ans: (a) Proven, (b) 2
4	(a) Find the values of p and q for which $\{x: x < -3$ or $x > 2\}$ is the solution set of $x^2 + q > -px$. (b) Find the range of values of m for which the line $y = 2x - \frac{3}{8}m$ meets the curve $y = 2x^2 + mx + 6$ at two distinct points. Ans: (a) $q = -6, p = 1$ (b) $m < -4$ or $m > 11$
5	(a)Find the smallest value of the integer a for which $ax^2 + 5x + 2$ is positive for all values of x. (b)Find the smallest value of the integer b for which $-5x^2 + bx - 2$ is negative for all values of x. Ans: $a = 4$, $b = -6$
6	Show that the roots of the quadratic equation $3(x + p)^2 - 1 = x - 1$ are not real if $p > \frac{1}{12}$. Ans: Shown
7	Prove that the line $y = x - 1$ always intersects the curve $y = nx^2 + 3x - n$ at 2 distinct points for all real values of n. Ans: Proven