7 MUST KNOW QUESTIONS TO CONQUER

BINOMIAL EXPANSION

1	(i) Expand $(1-2x)^9$ in ascending powers of x up to the term in x^3. (ii) Find the value of k, given that the coefficient of x in the expansion of $\left(3x+\frac{1}{kx^2}\right)(1-2x)^9$ is -53. Ans: (i) $(1-2x)^9 = 1-18x+144x^2-672x^3+\cdots$ (ii) $k=12$
2	(i) Expand $(3+2x)^8$, in ascending powers of x, up to the term in x^3. (ii) Write down the expansion of $(3-x)^3(3+2x)^8$ in ascending powers of x, up to x^2. (iii) By letting $x=0.01$ and your expansion in (iii), find the value of $2.99^3\times 3.02^8$, giving your answer correct to 3 significant figures. Show your workings clearly. Ans: i) $6561+34992x+81648x^2+108864x^3+\ldots$ ii) $177147+767637x+2854035x^2+\cdots$ iii) 185000
3	(i) Find the term independent of x in the expansion of $2x\left(2x-\frac{1}{x^2}\right)^8$. (ii) Given that the coefficient of x in the expansion of $\left(x+\frac{k}{2x^2}\right)^{16}$ is 4368, find the value of k. Ans: i) -3584 ii) $k=2$
4	(i) By considering the general term in the binomial expansion of $\left(kx-\frac{1}{x^3}\right)^7$, where k is a constant, explain why there are no even powers of x in this expansion. (ii) Given that the coefficient of the third term is thrice the coefficient of the second term, find the value of k. Ans: (i) Since the power of x is $7-4r=2(3-2r)+1$ will always be odd, there are no even powers of x for this expansion. (ii) $k=-1$

5	(i) Write down, and simplify, the first three terms in the expansion of $\left(1-\frac{x^2}{2}\right)^n$, in ascending powers of x, where n is a positive integer greater than 2. (ii) The first three terms in the expansion, in ascending powers of x, of $(2+3x^2)\left(1-\frac{x^2}{2}\right)^n$ are $2-px^2+2x^4$, where p is an integer. Find the value of n and of p. Ans: i) $1-n\left(\frac{x^2}{2}\right)+\frac{n(n-1)}{8}x^4+\cdots$ ii) $n=8, n=-1$(NA) , $p=5$
6	(i) Find the value of n, given that the coefficients of x^4 and x^6 in the expansion of $\left(1+\frac{1}{3}x^2\right)^n$ are in the ratio of 3: 2. (ii)Hence, find the coefficient of x^6 in the expansion of $(1-6x+9x^2)\left(1+\frac{1}{3}x^2\right)^n$. Ans: (a) (i) n = 8 (ii) $30\frac{2}{27}$
7	(i) Find the first 3 terms in the expansion of $\left(2-\frac{x}{4}\right)^n$ in ascending powers of x, where n is a positive integer greater than 2. Give the terms in their simplest forms. (ii) In the expansion of $(4+x)^2\left(2-\frac{x}{4}\right)^n$, there is no term in x^2. Find the value of n. Ans: (i) $2^n-\frac{n\times 2^n x}{8}+\frac{n(n-1)}{128}\times 2^n\times x^2+\cdots$ (ii) n = 1 (rejected) or n = 8