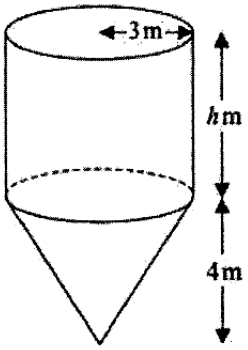
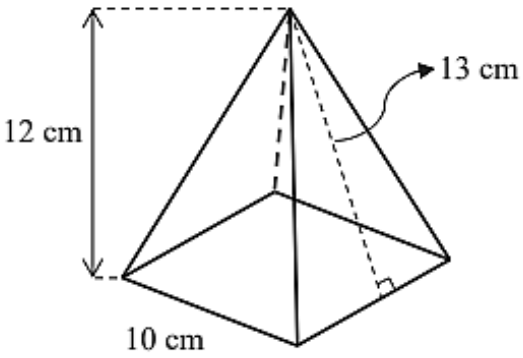
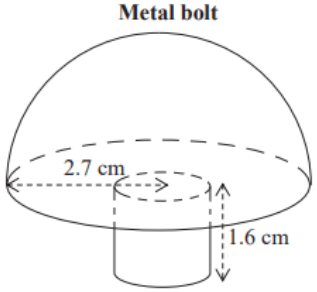
6 MUST KNOW QUESTIONS TO *CONQUER*

MENSURATION

1	A construction company use the following container for sand.  The container is made up of a cylinder on top of a cone. The cylinder has a radius of 3m and a height of *h* m. The cone has a base radius of 3m and a vertical height of 4m. The container is initially empty and is then filled with sand from the top at a constant rate. After 5 hours, the depth of the sand is 6m above the vertex of the cone. After 9 hours, the container is full of sand. Find the value of *h*. Give your answer as correct to two decimal places. You must show all your working.	[3]
2	 (a) A right pyramid has a vertical height of 12cm and a square base with sides of 10cm. Find its volume. (b) Given also that slants heights of the triangular faces are 13cm, find its total surface area.	[1] [2]

3



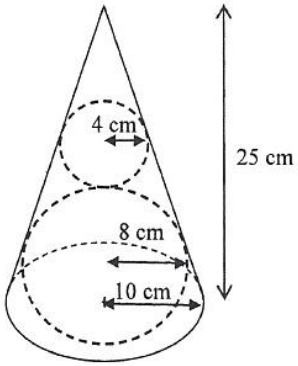
A metal bolt is made up of a hemisphere with a cylinder at the centre.
The height of the cylinder is 1.6 cm and the radius of the hemisphere is 2.7 cm.
The radius of the base of the cylinder is a third of the radius of the hemisphere.

(a) Calculate the volume of the metal bolt.
Give your answer correct to the nearest cubic centimetres. [3]

(b) A factory melts a big piece of metal to produce the bolts.
This piece of metal is in the shape of a cuboid with dimensions 1 m by 0.8m by 0.6 m. A renovation company wants to order 10 000 metal bolts from the factor. Is the big piece of metal sufficient to meet the order? [3]

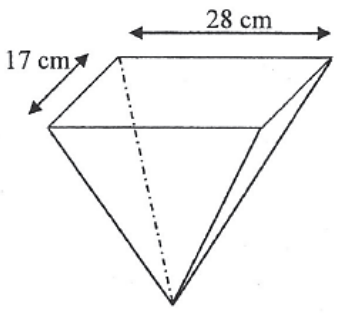
4

(a) 2 spherical metal balls, of radii 8 cm and 4 cm respectively is put into a regular conical container. The radius of the cone is 10 cm and the height of the cone is 25 cm.



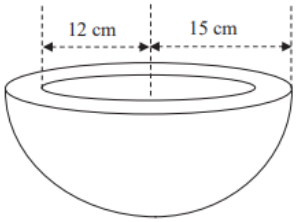
Calculate
(i) the volume of the 2 spherical metal balls,
(ii) the volume in the cone not occupied by the balls.

(b) The 2 spherical metal balls are melted and recast to form a solid rectangular pyramid as shown below.



Calculate the height of the solid pyramid.

5 A company wants to manufacture hollow hemispheric containers for sale. Each container has an external radius of 15 cm and an internal radius 12 cm. as shown in the diagram below.



(a) Taking $\pi = 3.142$, find the volume of the material that is needed to make each container.

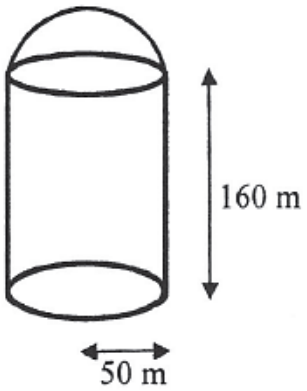
Leave your answer correct to 3 significant figures. [3]

(b) The company is evaluating 3 types of materials, *X*, *Y* and *Z* for manufacturing the container. The cost of each type of material is shown in the table below.

Material	*X*	*Y*	*Z*
Cost ($/cm^3)	0.0014	0.0021	0.0025

If the company wants the cost of each container to be less than \$15, determine the material/s suitable for making the containers. [3]

6 A new structure shown in the diagram below, has been built. It is made up of cylindrical bottom with height of 160 metres and a hemispherical top of radius 50 metres.



(a) Calculate the surface area of the hemispherical portion of the structure. [1]

(b) Find the volume of the structure. [3]

(c) The owner has set aside a budget of \$35000 to spruce up the structure that has been build. He is thinking of painting the cylindrical portion of the structure. If he is charged \$7.50 per m^3 for the painting services, would he have enough budget to proceed with the painting? Show your workings and explain clearly. [3]

Answer Key

1	Solution: $\frac{1}{3} \times \pi \times (3)^2 \times 4 = 12\pi$ $\pi \times (3)^2 \times (6-4) = 18\pi$ $12\pi + 18\pi = 30\pi$ $30\pi \times \frac{9}{5} = 54\pi$ $54\pi = 12\pi + \pi \times (3)^2 \times h$ $54\pi = 12\pi + 9\pi h$ $42\pi = 9\pi h$ $h = 4.67$ Ans: 4.67	
2	Solutions: (a) Volume $= \frac{1}{3} \times$ base area $\times$ height $= \frac{1}{3} \times 10 \times 10 \times 12$ $= 400\ \text{cm}^3$ (b) Base area $= 10 \times 10 = 100\ \text{cm}^2$ Lateral Surface/Arear of Triangle $= \frac{1}{2} \times 10 \times 13 = 65\text{cm}^2$ Total surface area$= 100 + 65 \times 4\ \text{cm}^2$ $= 360$ Ans: (a) 400 cm^3 (b) 360	
3	Solutions: (a) Volume $= \left(\frac{2}{3} \times \pi \times 2.7^3\right) + (\pi \times 0.9^2 \times 1.6)$ $= 45.295$ (b) $100 \times 80 \times 60 = 480000\ \text{cm}^3$ $= \frac{48000}{45}$ $= 10666$ Yes, it is sufficient. Ans: (a) 45 cm^3 (b) 10666. Yes, it is sufficient.	
4	Solutions: (a)(i) $\frac{4}{3}(3.142)(8)^3 + \frac{4}{3}(3.142)(4)^3$ $= 2413.056\text{cm}^2$ (ii) $\frac{1}{3}(3.142)(10)^2(25) - 2413.056$ $= 205\ \text{cm}^3$ Ans: (a)(i) 2413.056cm^2 (ii) 205 cm^3	

5 Solutions:

(a) Volume of material needed

$= \frac{1}{2} \times \frac{4}{3}\pi(15)^3 - \frac{1}{2} \times \frac{4}{3}\pi(12)^3$

$= 1098\pi$

$= 3449.9 \text{ cm}^3$

$\approx 3450 \text{ cm}^3$(3sf)

Alternate Method

Volume of big hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi(15)^3$

$= 7069.5 \text{ cm}^3$

Volume of hollow $= \frac{1}{2} \times \frac{4}{3}\pi(12)^3$

$= 3619.6 \text{ cm}^3$

Volume of material needed $= 7069.5 - 3619.6$

$= 3449.9 \text{ cm}^3$

$\approx 3450 \text{ cm}^3$(3sf)

(b) For material *X*,

cost $= \$0.0014 \times 3449.9$

$= \$4.83$

For material *Y*,

cost $= \$0.0021 \times 3449.9$

$= \$7.24$

For material *Z*,

cost $= \$0.0025 \times 3449.9$

$= \$8.62$

All 3 materials, *X*, *Y* and *Z* are suitable.

[M1] for showing the correct steps of working for all 3.

[M1] for getting the correct cost of all 3 types of materials using 5sf working.

[A1] for deriving the correct conclusion based on calculated values.

Ans: (a) 3450 cm^3(3sf) (b) For *X* = \$4.83, For *Y*= \$7.24 & For *Z*= \$8.62

6	Solutions: (a) $2\pi r^2 = 2 \times \pi \times 50^2$ $= 15707.96327$ $\approx 15700\ \text{m}^3$ (b) Volume of cylinder $= \pi \times 50^2 \times 160$ $= 1256637.061\ \text{m}^3$ Volume of hemisphere $= \frac{2}{3} \times \pi \times 50^3$ $= 261799.3878\ \text{m}^3$ Total Volume $= 1256637.061\text{m}^3 + 261799.3878\text{m}^3$ $\approx 1520000\text{m}^3$ (c) $\pi \times d \times h$ $= \pi \times 100 \times 160$ $= 5026.548246\ \text{m}^3$ $= 5026.548246 \times \$7.50$ $= \$37699.11184$ No he does not have enough budget to proceed with the painting Ans: (a) $15700\ \text{m}^3$ (b) 1520000m^3 (c) $= \$37699.11184$	