

7 MUST KNOW QUESTIONS TO CONQUER

HCF & LCM

1	(a) Written as a product of its prime factors, $400 = 2^x \times 5^y$. Find the values of x and y .	[2]
	(b) Is 400 a perfect cube? You may use your answer in part (a) to explain why.	[2]
2	Written as a product of its prime factors, $56 = 2^3 \times 7$. (a) Find k such that $56k$ is both a perfect square and a perfect cube. (b) Express 42 as a product of its prime factors. Give your answer in index notation. (c) Find the highest common factor 42 and 56. (d) Two alarm clocks are set to ring at intervals of 42 minutes and 56 minutes, respectively. If the alarm clocks ring together at 0830, at what time will they next ring together again?	[1] [1] [1] [2]
3	(a) Express 280 as a product of its prime factors. (b) Find the smallest possible integer value of n such that $280n$ is a multiple of 24.	[1] [1]
4	(a) Find the highest common factor of 1428 and 2660. (b) Find the lowest common multiple of $2^6 \times 3^2 \times 5^{15}$ and $2^5 \times 3^3 \times 7$. Leave your answer in index notation form. (c) Using prime factorisation, find $\sqrt{2304}$. Show your working clearly. (d) Find the smallest integer value of k where $168k$ is a perfect square.	[1] [1] [2] [2]
5	(a) Express 784 as a product of its prime factors. (b) Using your answer in (a) explain why 784 is a perfect square. (c) Find the largest integer that can divide both 784 and 32. (d) Given that $784m$ is a multiple of 42. Find the smallest possible integer value of m .	[1] [1] [1] [1]
6	The number 108, written as the product of its prime factors, is $2^2 \times 3^3$. (a) Express 234 as a product of its prime factors, giving your answer in index notation. (b) Find the smallest non-zero whole number that is divisible by both 108 and 234. (c) a and b are both prime numbers. Find the values of a and b so that $108 \times 234 \times \frac{a}{b}$ is a perfect cube.	[2] [1] [2]

7	<p>(a) Express 450 as the product of its prime factors.</p> <p>(b) Mr. Ng distributed 150 rulers, 450 pens, and 350 pencils equally among his students.</p> <p>(i) Calculate the largest possible number of students in his class.</p> <p>(ii) Find the number of rulers, pens and pencils that were given to each student.</p>	[1] [1] [1]
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Answer Key

1	<p>Solution: (a)</p> <table style="margin-left: 40px;"> <tr><td>2</td><td>400</td></tr> <tr><td>2</td><td>200</td></tr> <tr><td>2</td><td>100</td></tr> <tr><td>2</td><td>50</td></tr> <tr><td>5</td><td>25</td></tr> <tr><td>5</td><td>5</td></tr> <tr><td>1</td><td></td></tr> </table> $400 = 2^4 \times 5^2$ <p>Ans: (a) $x = 4$, $y = 2$ (b) 400 is not a perfect cube because the powers of the prime factors 2 and 5 are not multiples of 3.</p>	2	400	2	200	2	100	2	50	5	25	5	5	1		
2	400															
2	200															
2	100															
2	50															
5	25															
5	5															
1																
2	<p>Solution: (a) $k = 2^3 \times 7^5 = 134456$</p> <p>Ans: (a) 134456, (b) $42 = 2 \times 3 \times 7$, (c) 14, (d) 1118</p>															
3	<p>Ans: (a) $2^3 \times 5 \times 7$ (b) $n = 3$</p>															
4	<p>Solutions:</p> <p>(a) $1428 = 2 \times 2 \times 3 \times 7 \times 17$ $2660 = 2 \times 2 \times 5 \times 7 \times 19$</p> <p>(c) $2304 = 2^8 \times 3^2$ $\sqrt{2304}$ $\sqrt{2^8 \times 3^2}$</p> <p>(d) $168 = 2^3 \times 3 \times 7$ $168k = 2^4 \times 3^2 \times 7^2$ $k = 2 \times 3 \times 7$ $k = 42$</p> <p>Ans: (a) 28 (b) $2^6 \times 3^3 \times 5^{15} \times 7$ (c) 48 (d) $k = 42$</p>															
5	<p>Ans: (a) $2^4 \times 7^2$, (b) 784 can be expressed as $(2^2 \times 7)^2$ OR the powers of each of its prime factor is even/divisible by 2/is a multiple of 2. (c) 16, (d) 3</p>															

6 Solutions:

$$\begin{array}{r|l}
 2 & 234 \\
 \hline
 3 & 117 \\
 \hline
 3 & 39 \\
 \hline
 13 & 13 \\
 \hline
 & 1
 \end{array}$$

$$234 = 2 \times 3^2 \times 13$$

$$(b) 108 = 2^2 \times 3^3$$

$$234 = 2 \times 3^2 \times 13$$

$$\text{LCM of } 108 \text{ and } 234 = 2^2 \times 3^3 \times 13$$

$$= 1404$$

$$(c) 108 \times 234 \times \frac{a}{b} = \frac{2^3 \times 3^5 \times 13 \times a}{b}$$

For expression to be a perfect cube, the index of each prime factor must be a multiple of 3. Hence, $a = 3, b = 13$.

Ans: (a) $234 = 2 \times 3^2 \times 13$, (b) 1404, (c) $a = 3, b = 13$.

7 Solution:

$$(b)(i) 150 = 2 \times 3 \times 5^2$$

$$350 = 2 \times 5^2 \times 7$$

$$\text{HCF of } 150, 450 \text{ and } 350 \text{ is } 2 \times 5^2 = 50$$

$$\text{Largest possible number of students} = \mathbf{50}$$

Ans: (a) $450 = 2 \times 3^2 \times 5^2$, (b)(i) 50, (b)(ii) 3 rulers, 9 pens, and 7 pencils