Name:	School:	Target Grade:

PARADIGM

MOCK O LEVEL PAPER 2023
SECONDARY

READ THESE INSTRUCTIONS FIRST

INSTRUCTIONS TO CANDIDATES

1. Find a nice comfortable spot without distraction.

2. Be fully focused for the whole duration of the test.

3. Speed is KING. Finish the paper as soon as possible then return-back to Check Your Answers.

4. As you are checking your answers, always find ways to VALIDATE your answer.

5. Avoid looking through line by line as usually you will not be able to see your Blind Spot.

6. If there is no alternative method, cover your answer and REDO the question.

7. Give non-exact answers to 3 significant figures, or 1 decimal place for angles in degree, or 2 decimal place for $$$, unless a different level of accuracy is specified in the question.

Wish you guys all the best in this test.

You can do it.

I believe in you.

Team Paradigm

PARADIGM [Turn Over]

MATHEMATICAL FORMULAE

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta = \frac{1}{2}ab\sin C$$

A Maths O Level Mock Paper 2023

1	**[Nature of Roots]** (a) Find the range of values of k for which $(k-3)x^2+4x+k$ is always positive for all real values of x. (b) Show that the roots of the equation $6x^2+4(m-1)=2(x+m)$ are real if $m \leq 2\frac{1}{12}$.	[2] [2]
2	**[Surds]** (a) Express $\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2$ in the form $a+b\sqrt{15}$, where a and b are integers.	[2]
3	[**Exponential]** An object is heated until it reaches a temperature of T_0°C. It is then allowed to cool. Its temperature, T°C, when it has been cooled for n minutes, is given by the equation $T=33+12e^{-\frac{3}{4}n}$. (i) Find the value of T_0. (ii) Find the value of n when $t=37$°C. (iii) Find the value of n at which the rate of decrease of temperature is 0.67°C/minute. (iv) Explain why the temperature of the object is always greater than 33°C. (v) Sketch the graph of $T=33+12e^{-\frac{3}{4}n}$.	[1] [1] [1] [1] [1]
4	[**Logarithm]** **(a)** Find the value(s) of y that satisfy the equation $$\log_4(2y)=\log_{16}(y-3)+3\log_9 3$$ (b) Solve the equation $3\log_x 3=8-4\log_3 x$.	[3] [3]
5	**[Polynomial]** (a) Factorise completely $27a^3-125b^3$. (b) It is given that $3x^3+3x^2-11x-6$ when divided by $x+a$ has a remainder that is half the remainder when it is divided by $x-a$. (i) Show that $3a^3-a^2=11a-2$. (ii) Solve $3a^3-a^2=11a-2$, giving your answer to two decimal places where necessary. **[Partial Fraction]** (c) Express $\frac{4x^3+x^2+6}{(x-2)(x^2+2)}$ in partial fractions.	[2] [2] [3] [4]

6 **[Binomial Theorem]**

(a) By considering the general term in the binomial expansion of $\left(kx - \frac{1}{x^3}\right)^7$, where k is a constant, explain why there are no even powers of x in this expansion. [3]

(b) Given that the coefficient of the third term is thrice the coefficient of the second term, find the value of k. [3]

7 **[Circle]**

The equation of the tangent to a circle at the point $A(8, 9)$ is given by $4y + 3x = 60$. The line $y = 4x - 7$ passes through the centre, P, of the circle.

(a) Find the coordinates of P. [2]
(b) Find the equation of the circle. [2]
The tangent to the circle at A meets the y-axis at point B.
(c) Find the equation of another circle with BP as diameter. [2]

8 **[Linear Law]**

A cup of hot coffee was put on the table to cool at 3 pm. The temperature of the coffee, T °C, after x minutes, is given $T = 20 + ae^{-kx}$ where a and k are constants. The table shows that values of T and x taken at different timings. It is believed that an error was made in recording one of the values of T.

x	5	10	15	20
T	68.5	60.1	52.6	37.1

(a) Using a scale of 4 cm to 5 minutes for x and 4 cm to 1 unit for $\ln(T - 20)$, plot $\ln(T - 20)$ against x and draw a straight line graph. [2]
(b) Determine which value of T, in the table above, is the incorrect recording and use your graph to estimate its correct value. [1]
Use your graph to estimate,
(c) the value of a and the value of k. [2]
(d) the time when the temperature of the coffee is $50°C$. [1]
(e) Explain why the temperature of the coffee is always more than $20°C$. [1]

9 **[Trigonometry]**

(a) (i) Prove that $\frac{\sin x}{\sec x + 1} + \frac{\sin x}{\sec x - 1} = 2 \cot x$. [3]

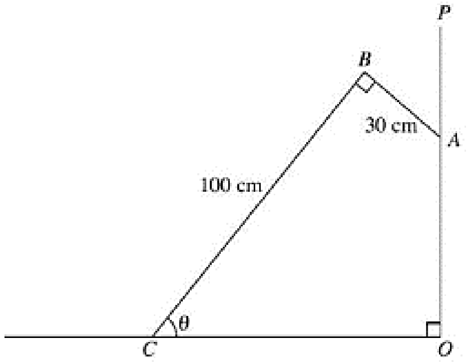
(ii) Hence find, for $0 \leq x \leq 4$, the exact solutions of the equation

$$\frac{\sin x}{\sec x + 1} + \frac{\sin x}{\sec x - 1} = \frac{2 \tan x}{3}$$ [3]

(b) Given that θ is obtuse and that $\sin\theta = \frac{1}{\sqrt{3}}$, express, **without the use of a calculator**, $\frac{1}{\sin\theta - \cos\theta}$ in the form $\sqrt{a} - \sqrt{b}$ where a and b are integers. [3]

10 **[R Formula]**

The figure shows a stage prop ABC used by a member of the theatre, leaning against a vertical wall OP. It is given that $AB = 30$ cm, $BC = 100$ cm, $\angle ABC = \angle AOC = 90°$ and $\angle BCO = \theta$.



(i) Show that $OC = (100\cos\theta + 30\sin\theta)$ cm. [2]

Let *D* be foot of *B* on *OC*, let *E* be foot of *A* on *BD*.

(ii) Express *OC* in terms of $R\cos(\theta - \alpha)$, where *R* is a positive constant and α is an acute angle. [2]

(iii) State the maximum value of OC and the corresponding value of θ. [2]

(iv) Find the value of θ for which $OC = 80$ cm. [2]

11 **[Differentiation – Equation of Tangent and Normal]**

The equation of a curve is $y = \frac{a}{x} + bx - 1$, where a and b are constants. The normal to the curve at the point $Q(1, -1)$ is parallel to the line $4y - x = 20$. This normal meets the curve again at point *P*.

(i) Find the value of a and of b. [4]

(ii) Find the coordinates of point *P*. [2]

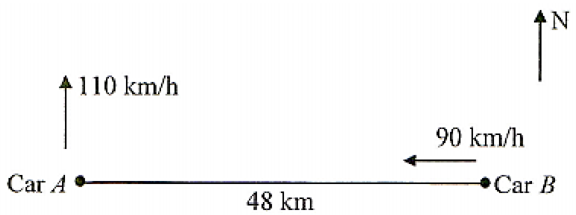
12 **[Differentiation – Rate of Change]**

The equation of a curve is given by $y = \ln\sqrt{\frac{5x}{9x+4}}$.

(i) Find $\frac{dy}{dx}$, expressing it as a single fraction. [2]

(ii) Find the rate at which *x* is changing when the graph crosses the *x*-axis, given that *y* is increasing at a rate of 0.3 units per second. [2]

13 **[Differentiation- Maxima & Minima]**



The diagram shows Car B, which is 48 km due east of Car A. Both cars start moving at the same time. Car A travels due north at a constant speed of 110 km/h while Car B travels due west at a constant speed of 90 km/h.

(i) The distance between Car A and Car B at time t hours after the cars started moving is denoted by L km. Express L in the form of $\sqrt{pt^2 + (q - rt)^2}$ where p, q and r are constants. [2]

(ii) Given that t can vary, find the stationary value of L. [2]

(iii) Determine whether this value stationary value of L gives the maximum or minimum distance between Car A and Car B. [2]

14 **[Integration – Hence (Differentiation)]**

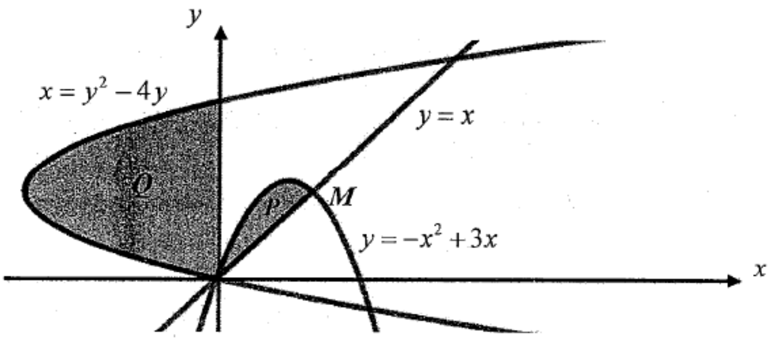
(a) Differentiate the following with respect to x.

(i) $\ln(\cos 2x)$ [2]

(ii) $\frac{x}{2}\tan 2x$ [2]

(b) Using your results from part **(a)**, find $\int 2x \sec^2 2x \, dx$. [2]

15 **[Integration – Area Under Graph]**



(a) M is the point of intersection of $y = x$ and $y = -x^2 + 3x$.
Show that the coordinates of M is $(2, 2)$. [2]

(b) Find the area P, bounded by the curve $y = -x^2 + 3x$ and the line $y = x$. [3]

(c) Find the area Q, enclosed by the curve $x = y^2 - 4y$ and the y-axis. [2]

16	**[Integration – Kinematics]** A particle starts from rest, travels in a straight line so that t is the time in seconds after passing a fixed point O. Its velocity, v m/s, is given by $v = 6t - 2t^2$. The particle comes to instantaneous rest at A. (i) Find the acceleration of the particle at A. (ii) Find the maximum velocity of the particle. (iii)Find the total distance travelled by the particle during the first 5 seconds.	[2] [2] [3]

Answer:

1	(a) $\therefore k > 4$ $(k-3)x^2 + 4x + k > 0$ for all values of x Discriminant < 0 $16 - 4k(k-3) < 0$ $4k^2 - 12k - 16 > 0$ $k^2 - 3k - 4 > 0$ $(k-4)(k+1) > 0$ $k > 4$ or $k < -1$ Since $k - 3 > 0, k > 3$ $\therefore k > 4$ (b) shown $6x^2 + 4(m-1) = 2(x+m)$ $6x^2 - 2x + 2m - 4 = 0$ Discriminant $= 100 - 48m$ Since $m \leq 2\frac{1}{12}$ $25 - 12m \geq 0$ $100 - 48m \geq 0$ Since discriminant ≥ 0, $6x^2 + 4(m-1) = 2(x+m)$ has real roots if $m \leq 2\frac{1}{12}$
2	(i) $31 + 8\sqrt{15}$ $\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2$ $= \frac{5+2\sqrt{15}+3}{5-2\sqrt{15}+3}$ $= \frac{8+2\sqrt{15}}{8-2\sqrt{15}} \times \frac{8+2\sqrt{15}}{8+2\sqrt{15}}$ $= \frac{64+32\sqrt{15}+60}{64-60}$ $= \frac{124+32\sqrt{15}}{4} = 31 + 8\sqrt{15}$

3

(i) Sub $n = 0, T_0 = 45°$

(ii) $n = 1.46$

$37 = 33 + 12e^{\frac{3}{4}n}$

$n = 1.46$

(iii) $n = 3.46$

$\frac{dT}{dn} = 12\left(-\frac{3}{4}\right)e^{-\frac{3}{4}}$

$-0.67 = -9e^{-\frac{3}{4}n}$

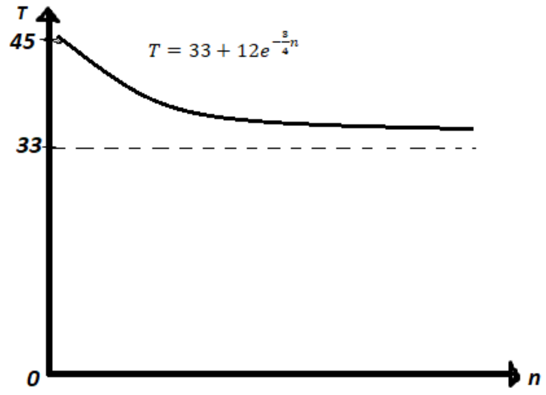
$n = 3.46$

(iv) shown

$12e^{-\frac{3}{4}n} > 0$

$33 + 12e^{-\frac{3}{4}n} > 33$

$T > 33℃$ (shown)



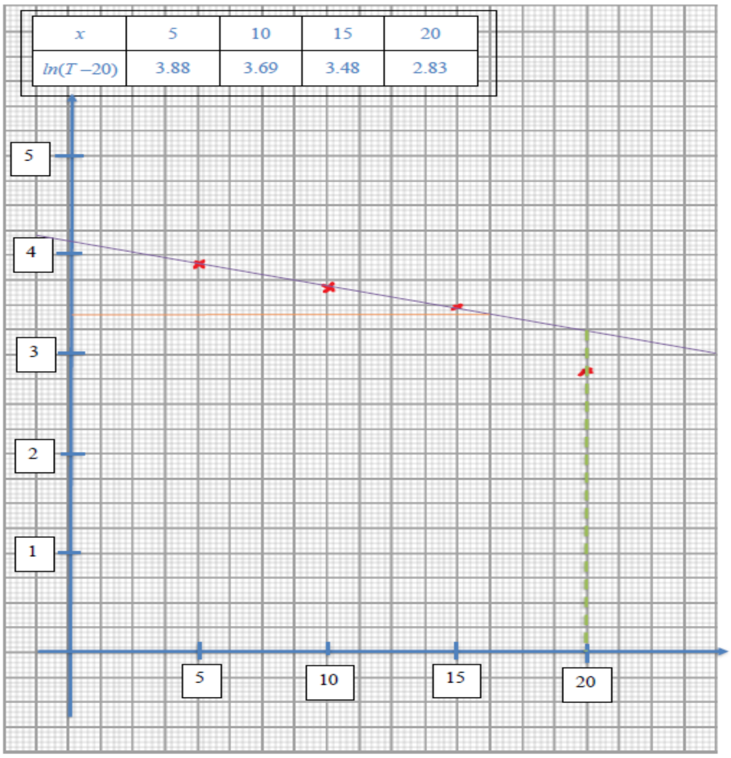
4	(a) $y = 4$ or $y = 12$ $\log_4(2y) = \log_{16}(y-3) + 3\log_9 3$ $\log_4(2y) = \frac{\log_4(y-3)}{\log_4 16} + 3\frac{\log_3 3}{\log_3 9}$ $\log_4(2y) = \frac{\log_4(y-3)}{2} + \frac{3}{2}$ $2\log_4(2y) = \log_4(y-3) + 3$ $\log_4(2y)^2 - \log_4(y-3) = 3$ $\log_4\frac{(2y)^2}{y-3} = 3$ $\therefore \frac{4y^2}{y-3} = 4^3$ $4y^2 = 64(y-3)$ $y^2 = 16(y-3)$ $y^2 - 16y + 48 = 0$ $(y-4)(y-12) = 0$ $\therefore y = 4$ or $y = 12$ (b) $\sqrt{27}$ *or* $\sqrt{3}$ $3\log_x 3 = 8 - 4\log_3 x$ $\frac{3}{\log_3 x} = 8 - 4\log_3 x$ *Let* $y = \log_3 x$ $\frac{3}{y} = 8 - 4y$ $3 = 8y - 4y^2$ $4y^2 - 8y + 3 = 0$ $(2y-3)(2y-1) = 0$ $y = 1.5$ *or* 0.5 $x = 3^{1.5}$ *or* $3^{0.5}$ $= \sqrt{27}$ *or* $\sqrt{3}$

5	(a) $(3a-5b)(9a^2+15ab+25b^2)$ $27a^3-125b^3$ $=(3a)^3-(5b)^3$ $=(3a-5b)[(3a)^2+(3a)(5b)+(5b)^2]$ $=(3a-5b)[9a^2+15ab+25^2]$ (bi) shown Let $f(x)=3x^3+3x^2-11x-6$ $f(-a)=3(-a)^3+3(-a)^2-11(-a)-6$ $=-3a^3+3a^2+11a-6$ $f(a)=3(a)^3+3(a)^2-11(a)-6$ [M1] $-3a^3+3a^2+11a-6=\frac{1}{2}(3a^3+3a^2-11a-6)$ $0=9a^3-3a^2-33a+6$ $0=3a^3-a^2-11a+2$ [M1] $3a^3-a^2=11a-2$ (bii) x =2, 0.18 or -1.85 $3a^3-a^2-11a+2=0$ $a-2$ is a factor. $(a-2)(3a^2+pa-1)=0$ $-2p-1=-11$ $2p=10$ $p=5$ $(a-2)(3a^2+5a-1)=0$ $a-2=0$ $3a^2+5a-1$ $a=2$ $a=\frac{-5\pm\sqrt{(5)^2-4(3)(-1)}}{2(3)}=0.18\ or-1.85\quad(2dp)$ (c) $\frac{4x^3+x^2+6}{(x-2)(x^2+2)}=4+\frac{7}{x-2}+\frac{2x-4}{x^2+2}$ $\frac{4x^3+x^2+6}{(x-2)(x^2+2)}=4+\frac{A}{x-2}+\frac{Bx+C}{x^2+2}$ Multiplying by $(x-2)(x^2+2)$, we obtain $4x^3+x^2+6=4(x-2)(x^2+2)+A(x^2+2)+(Bx+C)\,(x-2)$ Sub $x=2$: $\quad 4\times8+4+6=A(4+2)$ $42=6A\Rightarrow A=7$ Sub $x=0$: $\quad 6=-16+2(7)+C(-2)$ $-2C=8\Rightarrow C=-4$ Compare x^2: $1=-8+7+B\Rightarrow \quad B=2$ $\therefore\frac{4x^3+x^2+6}{(x-2)(x^2+2)}=4+\frac{7}{x-2}+\frac{2x-4}{x^2+2}$

6	(a) Since the power of x is $7-4r=2(3-2r)+1$ will always be odd, there are no even powers of x for this expansion. $\left(kx-\frac{1}{x^3}\right)^7=\binom{7}{r}(kx)^{7-r}\left(-\frac{1}{x^3}\right)^r+\dots$ $\left(kx-\frac{1}{x^3}\right)^7=\binom{7}{r}(k)^{7-r}(-1)^r x^{7-r}(x^{-3})^r+\dots$ $\left(kx-\frac{1}{x^3}\right)^7=\binom{7}{r}(k)^{7-r}(-1)^r x^{7-r-3r}+\dots$ $\left(kx-\frac{1}{x^3}\right)^7=\binom{7}{r}(k)^{7-r}(-1)^r x^{7-4r}+\dots$ Since the power of x is $7-4r=2(3-2r)+1$ will always be odd, there are no even powers of x for this expansion. (b) $k=-1$ $\binom{7}{2}(k)^{7-2}(-1)^2=3\binom{7}{1}(k)^{7-1}(-1)^1$ $21(k)^5=-3(7)(k)^6$ $k=-1$
7	(a) $P(2,1)$ $4y+3x=60$ $y=-\frac{3}{4}x+15$ $m_{\text{tangent}=-\frac{3}{4}}$ $m_{\text{normal}=\frac{4}{3}}$ $y-9=\frac{4}{3}(x-8)$ The equation of the normal is $y=\frac{4}{3}x-\frac{5}{3}$. -----(1) $y=4x-7$ --------(2) (1) = (2) : $\frac{4}{3}x-\frac{5}{3}=4x-7$ $x=2$ $y=1$ $P(2,1)$ (b) Equation of circle is $(x-2)^2+(y-1)^2=100$. $(x-2)^2+(y-1)^2=r^2$ Sub $(8,9)$: $r^2=100$ Equation of circle is $(x-2)^2+(y-1)^2=100$. (c) Equation of circle is $(x-1)^2+(y-8)^2=50$ When $x=0, 4y+3(0)=60$ $y=15$ $B(0,15)$ Centre of circle = $\left(\frac{2+0}{2},\frac{1+15}{2}\right)$ $=(1,8)$ $BP=\sqrt{(2-0)^2+(1-15)^2}$ $=10\sqrt{2}$ Radius = $5\sqrt{2}$ $(x-1)^2+(y-8)^2=\left(5\sqrt{2}\right)^2$ Equation of circle is $(x-1)^2+(y-8)^2=50$

8

(a) Graph



(b) 45.8

Incorrect value of T = 37.1

$$\ln(T-20) = 3.25$$

Correct value: $T - 20 = 25.790$

$$T = 45.79 \approx 45.8$$

(c) a = 63.4 , k = 0.045

$$T - 20 = ae^{-kx}$$

$\ln(T - 2 -) = \ln a - kx$

$Gradient = -k = \dfrac{4.15 - 3.25}{-20} = 0.045$

$k = 0.045[0.04 \le \ln a \le 4.15]$

$a = 63.4[57.4 \le a \le 63.4]$

(d) 3.17pm

$\ln(50 - 20) = \ln 30 = 3.40119$

From the graph, $x = 16.25[16.25 \le x \le 16.9]$

Time: 3.16pm or 3.17pm

(e) As T> 20, temperature of coffee will always be more than $20°C$.

$T - 20 = ae^{-kx}$

$T = 20 + ae^{-kx}$

As $ae^{-kx} > 0. T > 20$ and hence the lowest temperature of coffee always more than 20˚C.

9	(a)(i) Proved $LHS = \frac{\sin x}{\sec x+1} + \frac{\sin x}{\sec x-1}$ $= \frac{\sin x(\sec x-1)+\sin\ x(\sec x+1)}{\sec^2 x-1}$ $= \frac{\tan x-\sin x+\tan x+\sin x}{\sec^2 x-1}$ $= \frac{2\tan x}{\sec^2 x-1}$ $= \frac{2\tan x}{\tan^2 x}$ $= 2\cot x$ $= RHS$ (proved) (ii) $x = \frac{\pi}{3}, \frac{2\pi}{3}$ $\frac{\sin x}{\sec x+1} + \frac{\sin x}{\sec x-1} = \frac{2\tan x}{3}$ $2\cot x = \frac{2\tan x}{3}$ $\tan^2 x = 3$ $\tan x = \pm\sqrt{3}$ Basic angle = $\frac{\pi}{3}$ For $0 \leq x \leq 4$, $x = \frac{\pi}{3}, \frac{2\pi}{3}$ (b) $\sqrt{6} - \sqrt{3}$ $1^2 + x^2 = \left(\sqrt{3}\right)^2$ $x = \sqrt{2}$ $\cos\theta = -\frac{\sqrt{2}}{\sqrt{3}}$ $\frac{1}{\sin\theta-\cos\theta} = \frac{1}{\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{\sqrt{3}}}$ $= \frac{\sqrt{3}}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$ $= \frac{\sqrt{3}-\sqrt{6}}{-1}$ $= \sqrt{6} - \sqrt{3}$

10	(i) shown $\cos\theta = \frac{CD}{100} \Longrightarrow CD = 100\cos\theta$ $\sin\theta = \frac{AE}{30} \Rightarrow AE = 30\sin\theta$ $OC = CD + AE = 100\cos\theta + 30\sin\theta$ (ii) $\therefore OC = 10\sqrt{109}\cos(\theta - 16.7°)$ $R = \sqrt{100^2 + 30^2}$ $= 100\sqrt{109}$ $\alpha = \tan^{-1}\left(\frac{30}{100}\right)$ $= 16.7°(1dp)$ $\therefore OC = 10\sqrt{109}\cos(\theta - 16.7°)$ (iii) $OC_{max} = 10\sqrt{109}$ $\theta = 16.7°$ (iv) $\theta = 56.7°$ $80 = 10\sqrt{109}\cos(\theta - 16.7°)$ $co\,s(\theta - 16.7°) = \frac{8}{\sqrt{109}}$ $\theta - 16.7° = 39.98°(\theta\ is\ acute)$ $\theta = 56.7°$
11	(a) $a = 2, b = -2$ Equation of line: $y = \frac{1}{4}x + 5$ At $x = 1$, gradient of normal $= \frac{1}{4}$ Gradient of tangent $=-4$ $\frac{dy}{dx} = -\frac{a}{x^2} + b$ $-a + b = -4$ ----- Eqn (1) Sub $(1, -1)$ into $y = \frac{a}{x} + bx - 1$ $a + b = 0$ ---------- Eqn (2) Solving: $a = 2, b = -2$ (b) Coordinates of P is $\left(-\frac{8}{9}, -\frac{53}{36}\right)$ $y = \frac{2}{x} - 2x - 1$ Equation of normal is: $y + 1 = \frac{1}{4}(x - 1)$ $y = \frac{1}{4}x - \frac{5}{4}$ $\frac{2}{x} - 2x - 1 = \frac{1}{4}x - \frac{5}{4}$ $8 - 8x^2 - 4x = x^2 - 5x$ $9x^2 - x - 8 = 0$ $(9x + 8)(x - 1) = 0$ $x = -\frac{8}{9}$ or $x = 1$ $y = 2\left(-\frac{9}{8}\right) - 2\left(-\frac{8}{9}\right) - 1$ $= -\frac{53}{56}$ Coordinates of P is $\left(-\frac{8}{9}, -\frac{53}{36}\right)$

12

(a) $\frac{dy}{dx} = \frac{2}{x(9x+4)}$

$y = \ln\sqrt{\frac{5x}{9x+4}}$

$= \frac{1}{2}[\ln 5x - \ln(9x+4)]$

$\frac{dy}{dx} = \frac{1}{2}\left[\frac{5}{5x} - \frac{9}{9x+4}\right]$

$= \frac{1}{2}\left[\frac{9x+4}{x(9x+4)} - \frac{9x}{x(9x+4)}\right]$

$= \frac{1}{2}\left[\frac{4}{x(9x+4)}\right]$

$= \frac{2}{x(9x+4)}$

(b) x is increasing at a rate of $\frac{3}{4}$ units per second.

Let $y = 0$, $\ln\sqrt{\frac{5x}{9x+4}} = 0$

$\frac{1}{2}[\ln 5x - \ln(9x+4)] = 0$

$\ln 5x - \ln(9x+4) = 0$

$\ln 5x = \ln(9x+4)$

$5x = 9x + 4$

$x = -1$

$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$

$0.3 = \frac{2}{x(9x+4)} \times \frac{dx}{dt}$

When $x = -1$, $\frac{dx}{dt} = 0.3 \div \frac{2}{(-1)(-9+4)}$

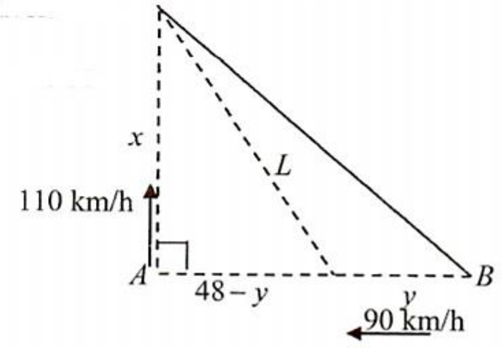
$= \frac{3}{4}$

x is increasing at a rate of $\frac{3}{4}$ units per second.

13

(i) $L = \sqrt{12100t^2 + (48 - 90t)^2}$

(i) Let the distance that Car A travels be x km.
Let the distance that Car B travels be y km.



For Car A,

$$110 = \frac{x}{t}$$

$$x = 110t\ km$$

For Car B,

$$90 = \frac{y}{t}$$

$$y = 90t\ km$$

By Pythagoras' Theorem,

$$L = \sqrt{x^2 + (48 - y)^2} \text{ (since L>0)}$$

$$= \sqrt{(110t)^2 + (48 - 90t)^2}$$

$$= \sqrt{(12100t^2 + (48 - 90t)^2}$$

(ii) 37.1

$$\frac{dL}{dt} = \frac{1}{2}[12100t^2 + (48 - 90t)^2]^2[24200t^2 + (48 - 90t)^2(-90)$$

$$= \frac{40400t - 8640}{2\sqrt{12100t^2 + (48 - 90t)^2}}$$

$$\frac{dL}{dt} = 0$$

$$40400t - 8640 = 0$$

$$t = \frac{108}{505}hr = 0.21386\ hr$$

$$L = \sqrt{12100\left(\frac{108}{505}\right)^2 + \left[48 - 90\left(\frac{108}{505}\right)\right]^2} = 37.06213 = 37.1(3\ s.f.)$$

(iii) Using the First Derivative Test,

t	0.21386^-	0.21386	0.21386^+
Sign of $\frac{dL}{dt}$	$-$	0	t
Sketch of tangent	\	—	/

Hence, L is minimum.

14	(a)(i) $-2\tan 2x$ $y = \ln(\cos 2x)$ $\frac{dy}{dx} = \frac{1}{\cos 2x} - \sin 2x\ 2$ $= -2\tan 2x$ (ii) $x\sec^2 2x + \frac{1}{2}\tan 2x$ $y = \frac{x}{2}\tan 2x$ $\frac{dy}{dx} = \frac{x}{2}\sec^2 2x\ 2 + \tan 2x\frac{1}{2}$ $= x\sec^2 2x + \frac{1}{2}\tan 2x$ (b) $\int 2x\sec^2 2x\,dx = x\tan 2x + \frac{1}{2}\ln\cos 2x + c$ $\int x\sec^2 2x + \frac{1}{2}\tan 2x\,dx = \frac{x}{2}\tan 2x$ $\int x\sec^2 2x\,dx = \frac{x}{2}\tan 2x - \frac{1}{2}\int \tan 2x\,dx$ $2\int x\sec^2 2x\,dx = x\tan 2x - \int \tan\ 2x\,dx$ $\int 2x\sec^2 2x\,dx = x\tan 2x + \frac{1}{2}\ln\cos 2x + c$
15	(a) x = 2, y = 2 $x = -x^2 + 3x$ $x^2 - 2x = 0$ $x(x-2) = 0$ $x = 0$ or $x = 2$ When $x = 2, y = 2$ Therefore M(2, 2) (b) $1\frac{2}{3}\ sq\ units$ $Area\ P = \int_0^2 (-x^2 + 2x)dx$ [M1] $= \int_0^2 (-x^2 + 2x)dx$ $= \left[-\frac{x^3}{3} + \frac{2x^2}{2}\right]_0^2$[M1] $= 4 - 2\frac{2}{3} = 1\frac{1}{3}units^2$ [A1] (c) $10\frac{2}{3}sq\ units$ $0 = y^2 - 4y$ $0 = y(y-4)$ $y = 0$ or $y = 4$ [M1] $Area\ P = \left\|\int_0^2 (y^2 - 4y)dy\right\| = \left\|\left[\frac{y^3}{3} - \frac{4y^2}{2}\right]_0^4\right\|$ [M1]=$10\frac{2}{3}units^2$ [A1]

16	(i) at A, $v = 0$ $\Rightarrow 2t(3-t) = 0$ $t = 0$ or $t = 3$ $a = \frac{dv}{dt}$ $a = 6 - 4t$ at A, acceleration $= 6 - 4(3) = -6$ ms (ii) For max velocity, $\frac{dv}{dt} = 0$ $6 - 4t = 0$ $t = \frac{3}{2}$ $\therefore$max velocity $= 6\left(\frac{3}{2}\right) - 2\left(\frac{3}{2}\right)^2 = 4\frac{1}{2} m/s$ (iii) $v = 6t - 2t^2$ $s = \int v\, dt$ $= \int 6t - 2t^2\, dt$ $= 3t^2 - \frac{2}{3}t^3 + c$ $when\ t = 0, s = 0 \Rightarrow c = 0$ $\therefore s = 3t^2 - \frac{2}{3}t^3$ when $t = 3$, $s = 3(3)^2 - \frac{2}{3}(3)^3 = 9$ when $t = 5$, $s = 3(5)^2 - \frac{2}{3}(5)^3 = -8\frac{1}{3}$